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New correlations for mixed turbulent natural and forced convection heat transfer in vertical tubes

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Abstract—This work deals with mixed natural and forced turbulent convection heat transfer in vertical tubes. It summarizes the experimental results available in literature for both aiding and opposing flow conditions and presents own experimental results that concentrated on the influence of length-to-diameter ratio and of heat and mass flux directions on heat transfer in vertical tubes. Finally, it presents two new empirical correlations that represent experimental results better than all existing correlations in literature, since they take into account the laminarization of turbulent flow that occurs during aiding mixed convection. They predict experimental results from literature and from this work with an accuracy of better than $\pm 20\%$. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

During the past 60 years numerous investigators have dealt with the influence of natural convection on heat transfer in vertical tube flows. There have been experimental and numerical studies investigating heat transfer coefficients, and temperature and velocity profiles in vertical tube flows under conditions where natural convection has significant impact on forced convection flow. Throughout the past four decades, papers have been published summarizing experimental and numerical results and theoretical considerations. Summaries have been published by Metais in 1963 [1], by Petukhov in 1978 [2] reviewing Russian studies, by Aung in 1987 [3], and by Jackson in 1989 [4], who reviewed numerous works by others. Some arrived at correlations more or less useful to describe mixed convection heat transfer. Such correlations were proposed, for example, by Churchill, 1977, [5], Petukhov, 1977, [6], Polley, 1981, [7], Swanson and Catton, 1987, [8], Jackson *et al.*, 1989, [4] and Joye, 1996, [9, 10]. They will be discussed in detail in the next section of this study. Just as a brief conclusion, they all have in common the fact that they do not describe the experimental results very well.

The aim of this work is to present experimental results for mixed convection heat transfer with special emphasis on the direction of heat and mass fluxes, and with respect to the length-to-diameter ratio of the tubes. The influence of both parameters has not been analyzed systematically so far. Finally, two correlations are presented which describe the experimental results of this and most published studies with an accuracy of better than $\pm 20\%$.

2. PREVIOUS WORK

The driving force for single-phase forced convection is a pressure gradient. With non-isothermal flows, temperature gradients induce density gradients which also act as a driving force for fluid motion in the presence of a gravitational field. In the event that density gradients are the only driving force, the flow is called pure natural convection.

Since fluid flow in heat exchangers is not isothermal, the impact of natural convection on forced convection heat transfer can be significant for temperature differences and mass flow velocities found usually. However, no useful heat transfer correlations exist. To disregard the influence of natural convection in vertical tube-and-shell heat exchangers may be fatal since it can increase or decrease heat transfer coefficients by up to one order of magnitude.

In principle, there are two ways of combining natural and forced convection in a vertical tube. When both driving forces act in the same direction it is called aiding flow, in different directions opposing flow. When it comes to thermal boundary conditions of channel flows, there are two theoretical limiting cases, the first being uniform wall temperature (UWT), the second being uniform heat flux (UHF).

The first experimental investigation of mixed turbulent convection heat transfer dates to 1939, when Watzinger and Johnson [11] measured heat transfer coefficients in turbulent water flows under both aiding and opposing flow conditions. The length-to-diameter ratio, the L/d ratio, was 20 for their test tube. The next who report experimental data are Norris and Sims in 1942 [12]. Since they investigated the heat

NOMENCLATURE

a	constant in equation (9)
A	heat transfer area [m^2]
b	constant in equation (9)
c	constant in equation (9)
c_p	specific heat at constant pressure [J (kg K)$^{-1}$]
d	inner diameter of the tube [m]
Gz	Graetz number
k	heat transfer coefficient [W (m^{-2} K^{-1})]
L	tube length [m]
$\dot{M}$	mass flow rate [kg s^{-1}]
Nu	Nusselt number
P	parameter, see equation (8)
Pr	Prandtl number
$\dot{Q}$	heat flow rate [W]
Ra	Rayleigh number
Re	Reynolds number
ΔT	temperature difference [K]
u	flow velocity [m s^{-1}].

Greek symbols

γ	parameter in equation (3)
ξ	friction factor.

Indices

A	aiding
ε	at the rim of the viscous layer
FL	forced laminar convection
FT	forced turbulent convection
in	inlet
log	logarithmic mean
m	mean
M	mixed convection
NT	natural turbulent convection
O	opposing
out	outlet
T	tube-side
w	wall.

transfer in oil flows under conditions where forced convection was the dominant driving force, their results will not be mentioned any further in this work. In 1957, Metais [13] published experimental data for heat transfer to water flowing in a tube with $L/d = 29$ with aiding natural and forced convection. In 1972, Herbert and Sterns [14] offered experimental data obtained from a study of aided and opposed convection heat transfer in a $L/d = 80$ tube. Axcell and Hall [15] in 1978 first presented data for air flows under opposed flow conditions. They used a tube with $L/d = 6$. Saylor and Joye, 1988 [16], performed measurements of heat transfer in turbulent water flows with aided and opposed influence of natural convection. The L/d-ratio of their tube was 50. At the authors' research institute, experiments of mixed convection heat transfer to turbulent water flow in a $L/d = 74$ tube were performed in 1989 by Temu *et al.* [17]. The experimental data from the papers cited above will be compared to our own results and to the correlations presented at the end of this study. Numerous experimental studies have been published, but could not be used, since they only give diagrams with curves, but no tables with experimental data. Table 1 contains a summary of the works from literature used in this study. It shows the authors' names, the year of publication, the L/d-ratio of the test tube, the direction of heat and mass fluxes, and the thermal boundary condition.

Except two authors, all findings show that heat transfer deteriorates for the two limiting cases of pure natural and pure forced convection under aiding flow conditions. For opposed combination, heat transfer ameliorates, again compared to the two limiting cases. Only the works of Watzinger and Johnson [11] and Metais [13] predict an increase in heat transfer in both opposed and aided mixed convection, for unknown reason.

Figures 1 and 2 schematically show the influence of natural convection on forced convection heat transfer for aiding and opposing flow, according to the majority of all studies as stated above. In both figures, the Nusselt numbers for mixed convection are related to the Nusselt number calculated for pure forced convection at the same Reynolds number. The related Nusselt numbers are plotted versus the parameter $Ra^{0.333}/(Re^{0.8} \cdot Pr^{0.4})$. Herein, the Rayleigh number characterizes the strength of the buoyancy forces and the Reynolds number the forced convection driving force. This parameter has been derived by comparing thermal boundary layers of pure natural to pure forced convection following an approach to mixed laminar convection by Bejan [18]. For natural convection the thickness of the thermal boundary layer is inversely proportional to $Ra^{0.333}$, for pure forced convection, it is inversely proportional to $(Re^{0.8} \cdot Pr^{0.4})$. Therefore, the parameter $Ra^{0.333}/(Re^{0.8} \cdot Pr^{0.4})$ enables statements about the significant driving force for given Rayleigh and Reynolds numbers. If this parameter exceeds 0.2, natural convection is the dominant driving force for the fluid motion; if less than 0.05, it is the forced convection. Figure 1 shows the decline of heat transfer for aiding natural and forced convection. The limiting cases are indicated by dotted lines. In Fig. 2 the behavior of heat transfer is demonstrated for opposed mixed convection.

This behavior, different for aiding and opposing flows, can be explained when visualizing the effect of natural convection on the velocity profile of a pure forced turbulent flow. In Fig. 3, velocity profiles are

Table 1. Summary of studies from literature used in this work. DC = downward flow cooled; DH = downward flow heated; UC = upward flow cooled; UH = upward flow heated; UWT = uniform wall temperature

Authors	Reference	L/d	Heat and mass flux direction	Thermal boundary condition
Watzinger and Johnson	1939 [11]	20	DC	UWT
Norris and Sims	1942 [12]	220	DC	UWT
Metais	1957 [26]	29	UH	UWT
Herbert and Sterns	1972 [4]	80	UH, DH	UWT
Axcell and Hall	1978 [15]	6	DH	UWT
Saylor and Joye	1988 [16]	50	DH	UWT
Temu *et al.*	1989 [17]	74	UH	UWT

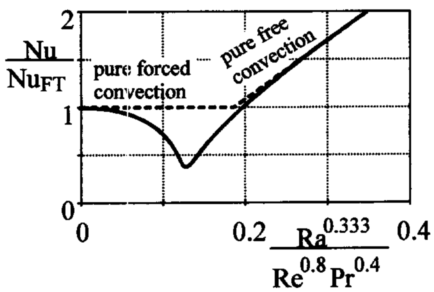

Fig. 1. Schematic diagram showing heat transfer for aiding mixed convection.

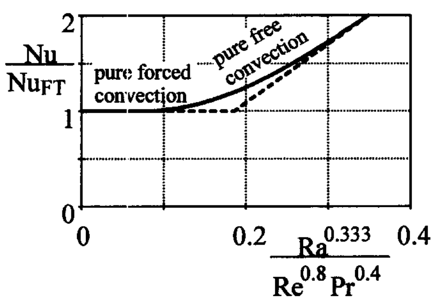

Fig. 2. Schematic diagram showing heat transfer for opposing mixed convection.

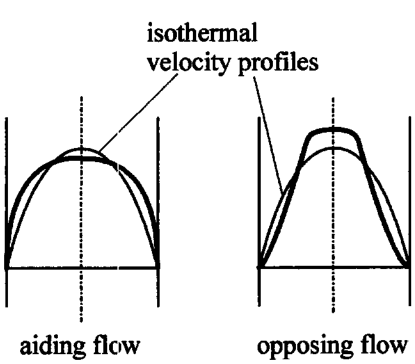

Fig. 3. Velocity profiles under aiding and opposing turbulent flow conditions.

shown schematically for both aiding and opposing mixed convection. They were determined experimentally i.a. by Steiner in 1971 [19], who measured turbulent flow velocity profiles by hot-wire anemometry for air flow under aiding convection conditions. Opposed turbulent velocity profiles in an air flow were determined by Cotton and Jackson in 1990 [20].

As can be seen in Fig. 3, if both natural and forced convection act in the same direction, the fluid is accelerated in the region close to the tube wall. Because of mass continuity, the flow is retarded in the core. Therefore, the velocity profile becomes steeper close to the walls and flattens in the center. The maximum flow velocity lies under the maximum in case of isothermal, i.e. pure forced, turbulent flow. The situation is vice versa for opposed mixed convection, the velocity profile is not as steep close to the walls, since the fluid is retarded by the impact of natural convection and higher in the core for mass continuity reason.

By means of this insight in natural convection changing velocity profiles, its influence on heat transfer can be understood qualitatively. According to Prandtl's model [21], heat transfer to turbulent flows is controlled by two mechanisms in series, i.e. heat conduction through the viscous layer (colored gray in Fig. 4) and diffusive energy transport from the border of the viscous layer to the core of the turbulent flow, with the latter being the significant one. The diffusive energy transport is proportional to the turbulence production in the area close to the rim of the viscous layer. The turbulence production depends on the difference between the velocity at the rim of the viscous layer, u_ε in Fig. 4, to that in the core of the flow, u_m in Fig. 4.

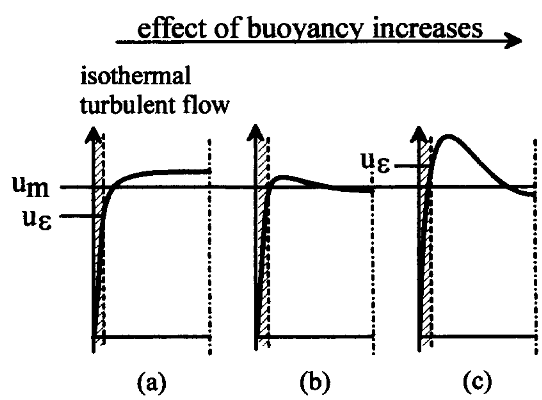

Fig. 4. Effect of aiding natural convection on a turbulent flow in a vertical tube. u_ε is the velocity at the border of the viscous layer, u_m is the mean velocity in the core of the flow.

The influence of aiding natural convection on turbulent forced convection is depicted in Fig. 4. On the left side, an isothermal velocity profile is shown. Moving to the middle and the right drawing shows what happens to the velocity profile when the buoyancy force arises and increases. It can be seen (Fig. 4(b)) that the absolute of the velocity difference in the core and the rim of the viscous layer becomes zero when increasing the impact of natural convection on pure forced convection flows. Further increase of the buoyancy forces (see Fig. 4(c)) enhances this difference. Since it determines the turbulence production, heat transfer shows its characteristic decline, when the influence of natural convection is augmented, before it rebounds with further increase of buoyancy force, see also Fig. 1. The reduced turbulence production for small buoyancy influence in aiding flow causes a laminarization of turbulent flows. Measurements of axial wall temperature distribution for UHF thermal boundary conditions show that local temperature maxima occur which indicate minima in heat transfer caused by the laminarization of the flow [4, 22]. Laminarization fully develops after a certain entrance length, reaches full strength, yielding the minimum heat transfer rate, and is overridden by increasing strength of buoyancy force.

For opposed mixed convection on the other hand, the difference between the velocity at the rim of the viscous layer and the core increases with increasing buoyancy force and yields a higher heat transfer (see Fig. 2). Laminarization is also stated by Jackson and Hall [23], and Axcell and Hall [15]. According to them, the natural convection brings about a change in shear stress distribution. In case of aiding mixed convection, the shear stress reduces when moving away from the tube wall to the center because in the near-wall region the buoyancy force and the force driving forced convection act in the same direction. This yields a smaller shear stress, which causes the turbulence production to decrease.

Numerical investigations by Tanaka *et al.* [24] using the k-ε-model reveal the same decrease in heat transfer for low Reynolds numbers, attributed by the authors to a decrease in the production of turbulence which is caused by a decrease in shear stress following laminarization. Laminarization also can be brought about by rotating the tube wall and thus the velocity difference between the core and rim of the viscous layer is decreased which leads to a reduction of heat transfer [25].

There are numerous papers approaching mixed convection heat transfer numerically. A very good summary is given by Jackson *et al.* [4]. In general, numerical results are in good agreement to experimental observations, although no correlations to predict heat transfer coefficients were presented by the researchers.

A first step to predict heat transfer coefficients was done by Metais and Eckert in 1963 [26]. They published maps showing the regions of natural, forced and mixed convection regimes with respect to Reynolds and Rayleigh numbers. Since they do not give heat transfer correlations in their publication, it will only be used to determine whether a certain combination of Reynolds and Rayleigh number yields a limiting case, i.e. pure natural or pure forced convection, respectively, or the mixed convection regime. At least for the pure forced convection limiting case, there do exist correlations in literature.

A promising way to correlate mixed convection heat transfer was pursued by Churchill [5, 27, 28] suggesting superposition of the two limiting cases. Unfortunately, when it comes to aided mixed turbulent flow, the correlation can yield Nusselt numbers of zero, which is clearly contradictory to experimental observations. In 1977, Petukhov [6] suggested multiplying the pure forced convection Nusselt numbers with a function that takes into account the buoyancy force and 4 years later Polley [7] came up with a slight modification of Petukhov's approach, suggesting another correlation for the pure forced convection Nusselt number in Petukhov's equation. The weak point of Petukhov's and Polley's equations is that they do not match the natural convection limiting case. A different approach based on the aforementioned shear stress study has been published by Jackson and Fewster [29]. Their theoretical considerations lead to two other superposition type correlations for aided and opposed flow. Swanson and Catton developed a similar equation for opposed convection flow in channels with rectangular cross-section [8]. The equations of both authors encounter the same deficiency as Churchill's equations. In 1996, Joye [9, 10] published two studies that concentrated on aiding and opposing mixed convection heat transfer in the region of Reynolds numbers from 700 to 25 000 and Grashof numbers between 10^7 and 10^8. He found that heat transfer tends to the pure laminar convection limiting case for Reynolds numbers below 4000 and the pure turbulent convection limiting case for Reynolds numbers above 10 000, but he does not offer a new correlation for the investigated Reynolds number range.

3. EXPERIMENTAL

Three different tubes made of copper, designed as double-pipe heat exchangers, were tested. Their dimensions are listed in Table 2. The assembly was mounted vertically with water circulating through the tube and the annulus. To provide constant wall temperature (UWT), the water in the annulus was circulated with a velocity high enough to keep its temperature change small compared to that of the water flow in the inner tube.

Water temperatures were measured using Ni–Cr/Ni thermocouples mounted at the inlet and outlet nozzles of both the tube and the shell side. To obtain caloric mean temperatures, static mixers were installed upstream of each thermocouple. The test results are based on the temperature change of the tube side flow

Table 2. Dimensions of the test double-pipe heat exchangers

Tube label	Length	Shell tube Outer diameter	Shell tube Inner diameter	Test tube Outer diameter	Test tube Inner diameter	L/d ratio
	[mm]	[mm]	[mm]	[mm]	[mm]	[-]
A5030	2000	53	50	30	27	74
A5040	2000	53	50	40	37	54
C5040	920	53	50	40	37	25

rather than the temperature change of the shell side fluid since the latter varied in the range of the accuracy of temperature measurements. Mass flow rates of both fluid flows were controlled by valves and measured by flowmeters. The experimental set-up is shown schematically in Fig. 5. It consists of two closed loops through which water was circulated. The water flow in the left loop was kept at a low temperature by means of a plate heat exchanger cooled with tap water. The right water loop is heated in another plate heat exchanger operated with steam at a maximum pressure of three bars. Either the cold or the hot water could be pumped through the inner tube in both directions, upwards and downwards, thus enabling all four combinations of upward or downward flow heating or cooling.

Tube-side Reynolds numbers were kept at constant values of 4500, 7500, 11 500 and 15 500, with Rayleigh numbers varying between 10^6 and 10^9. The Reynolds number is defined with the tube diameter as characteristic length and fluid properties at bulk temperature of the tube-side water flow, i.e. arithmetic mean of the inlet and outlet temperature of the tube-side flow, whereas the Rayleigh number uses film temperature for the fluid properties. The film temperature is the arithmetic mean of the bulk temperature and the mean wall temperature. The Reynolds number in the shell side was kept at 30 000–50 000, limiting temperature changes of the shell-side fluid of less than 0.3 K.

From the temperatures and the mass flow rates measured, overall heat transfer coefficients could be calculated according to

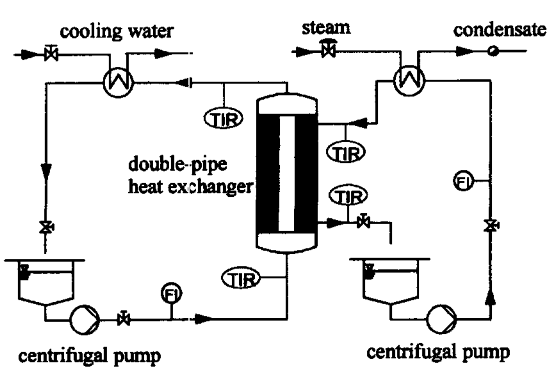

Fig. 5. Schematic diagram of the experimental set-up.

$$\dot{Q} = \dot{M}_T \cdot c_{p,T} \cdot |T_{T,in} - T_{T,out}| = k \cdot A \cdot \Delta T_{log} \quad (1)$$

where $\dot{Q}$ denotes the heat flow rate in the heat exchanger, $\dot{M}$ the mass flow rate in the test tube, $c_{p,T}$ the heat capacity of the fluid in the tube taken at bulk temperature and $T_{T,in}$ and $T_{T,out}$ the temperatures at the inlet and the outlet of the test tube. k is the heat transfer coefficient, A the area of the inner surface of the test tube and ΔT_{log} is the logarithmic mean temperature difference.

To derive the tube-side heat transfer coefficient, the shell-side and the tube wall resistance must be known. The first had been calculated using a correlation for heat transfer in an annulus taking into account nozzle effects [30], the latter was obtained by calculating heat conduction through a cylindrical tube wall. For further details see Kim [31] and Aicher and Kim [30].

4. EXPERIMENTAL RESULTS AND DISCUSSION

Throughout this chapter, mixed convection heat transfer will be expressed by related Nusselt numbers plotted over the parameter $Ra^{0.333}/(Re^{0.8} \cdot Pr^{0.4})$. Nusselt numbers experimentally obtained in this work or evaluated from data taken from literature are related to Nusselt numbers calculated for pure forced convection at the same Reynolds number according to the equation of Gnielinski [32].

- For $Re \geqslant 10\,000$ the Nusselt number for pure forced convection heat transfer is

$$Nu_{FT} = \frac{\xi/8 \cdot Re \cdot Pr}{1 + 12.7 \cdot \sqrt{\xi/8} \cdot (Pr^{2/3} - 1)} \cdot \left(1 + \left(\frac{d}{L}\right)^{2/3}\right) \quad (2)$$

with the friction factor calculated according to

$$\xi = (1.8 \cdot \log[Re] - 1.5)^{-2}.$$

- And for $2300 < Re < 10\,000$ it is

$$Nu = (1 - \gamma) \cdot Nu_{FL}(Re = 2300) + \gamma \cdot Nu_{FT}(Re = 10\,000) \quad (3)$$

with the parameter γ defined as

$$\gamma = \frac{Re - 2300}{10\,000 - 2300}.$$

In equation (3) the Nusselt number for pure forced laminar convection heat transfer at $Re = 2300$ is calculated according to [33]

$$Nu_{FL} = \left[3.66^3 + 0.7^3 + [1.615 \cdot \sqrt[3]{Gz} - 0.7]^3 + \left[\left(\frac{2}{1+22 \cdot Pr}\right)^{1/6} \cdot \sqrt{Gz}\right]^3\right]^{1/3} \quad (4)$$

with the Graetz number defined by

$$Gz = Re \cdot Pr \cdot d/L$$

and the Nusselt number for turbulent convection heat transfer at $Re = 10\,000$ is calculated using equation (2).

The factor $(Pr/Pr_w)^{0.11}$ originally in equations (2) and (3), that takes into account the temperature dependency of the viscosity (see [32]), has been omitted. The data taken from literature were re-evaluated the same way we treated our own experimental data, and added to the figures. All dimensionless groups are taken at bulk temperature, except the Rayleigh number, which is taken at film temperature, being the arithmetic mean of the bulk temperature and the mean wall temperature. The thermal boundary conditions UWT and UHF do not have to be distinguished, since in case of fluids with Prandtl numbers greater than 0.7 there is no significant difference between both [34, 35]. In this paper only data of UWT measurements were evaluated.

4.1. *Impact of heat flux direction on heat transfer*

For both combinations, aiding and opposing convection, there are two possibilities of heat and mass flux direction. Under aiding flow conditions, the fluid can either be heated when flowing upwards (upward heated = UH) or be cooled when flowing downwards (downward cooled = DC), provided that the fluid has a positive thermal expansion coefficient which is the case for the vast majority of all fluids. Opposed flow conditions can be obtained when cooling upward flow (upwards cooled = UC) or heated downward flow (downwards heated = DH). It is alleged in literature that there is no difference between the two aiding flows and the two opposing flows, respectively, but no systematic study has been published so far. Consequently, the first part of our work concentrates on this gap. Experiments were carried out at three different tube-side Reynolds numbers varying the tube-side Rayleigh number for all four combinations of heat and mass flux directions, i.e. UH, DC, UC and DH. In Figs. 6 and 7 our experimental results are shown for aiding and opposing flow for the test tube A5030 with $L/d = 74$ at Reynolds numbers of 7500, 11 500 and 15 500 and Rayleigh numbers between $6 \cdot 10^6$ and $9 \cdot 10^7$. For comparison experimental results of Herbert and Sterns [14] and Temu [17] for aiding flow are added because they used a tube with similar L/d-ratio.

As can be seen from Figs. 6 and 7 there is close

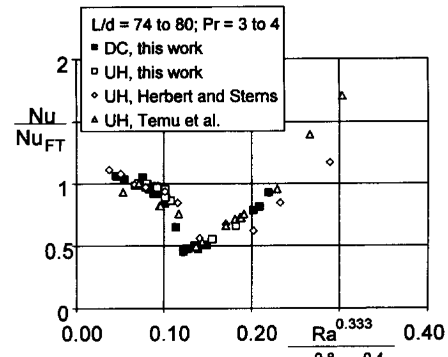

Fig. 6. Nusselt numbers related to Nusselt number for pure forced convection plotted over the parameter $Ra^{0.333}/(Re^{0.8} \cdot Pr^{0.4})$ for aiding mixed convection flow in vertical tubes with respect to the direction of heat flux. The experimental data of Herbert and Sterns [14] and Temu [17] are added. $L/d \cong 74$, $Pr \cong 3, 5$.

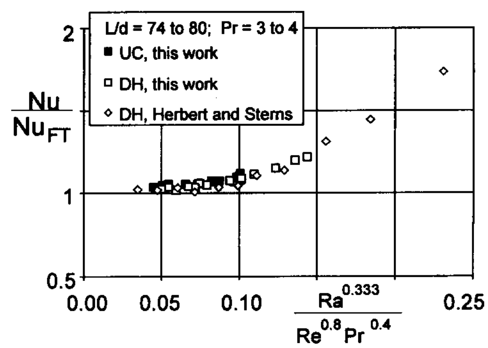

Fig. 7. Nusselt numbers related to Nusselt number for pure forced convection plotted over the parameter $Ra^{0.333}/(Re^{0.8} \cdot Pr^{0.4})$ for opposing mixed convection flow in vertical tubes with respect to the direction of heat flux. The experimental data were taken from this work and from Herbert and Sterns [14]. $L/d \cong 74$, $Pr \cong 3, 5$.

agreement between our own work and other researchers. As explained above, heat transfer expressed by the related Nusselt number decreases compared to the two limiting cases when aiding mixed convection is driving the fluid motion (see Fig. 6), whereas opposed combination leads to heat transfer improvement (see Fig. 7). As a result, in both cases, i.e. aiding and opposing flow, no significant difference between the two possible ways of combining heat and mass flux can be observed.

4.2. *Influence of length-to-diameter ratio*

Again, no researcher seems to have dealt with the L/d impact on heat transfer in mixed convection. Because of this, experiments with three tubes differing in the L/d-ratio were performed under aiding and opposing flow conditions for upward mass flow direction, which leads to UH and UC combination. Under aiding flow conditions, heat transfer experiments with tubes of $L/d = 25$, 54 and 74 were performed, and under opposed flow conditions with tube of 25 and 54. Again, the results expressed by the related Nusselt

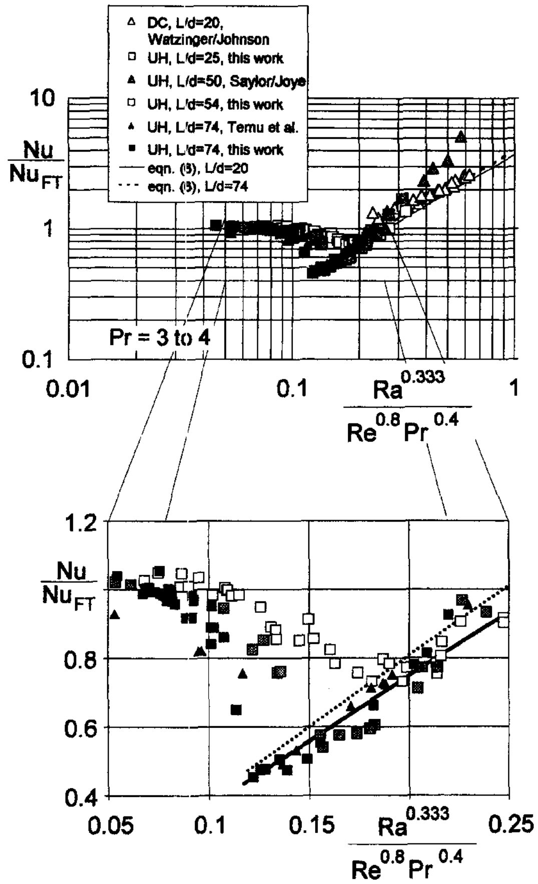

Fig. 8. Nusselt numbers related to Nusselt number for pure forced convection plotted over the parameter $Ra^{0.333}/(Re^{0.8} \cdot Pr^{0.4})$ for aiding mixed convection flow in vertical tubes with respect to L/d ratio. Data were taken from this work and from literature [11, 16, 17]. $Pr \cong 3, 5$.

numbers are plotted vs the parameter $Ra^{0.333}/(Re^{0.8} \cdot Pr^{0.4})$ in Figs. 8 and 9 for aiding and opposing flow conditions. For comparison related Nusselt numbers evaluated from the experimental data of Watzinger and Johnson, 1939 [11], Herbert and Sterns, 1972 [14], Axcell and Hall, 1978 [15], Saylor and Joye, 1988 [16] and Temu 1989 [17], are added to the diagrams. For aiding flow, Fig. 8 again shows the characteristic decline in heat transfer for conditions where natural and forced convection act together. The limiting case of pure forced convection, for $Ra^{0.33}/(Re^{0.8} \cdot Pr^{0.4}) < 0.05$, can be stated. To demonstrate the natural convection limiting case, the correlation for pure turbulent natural convection derived later in this paper, see equation (6), is added to the figure as solid and dotted lines for L/d for 20 and 74. The agreement with the data of Watzinger and Johnson [11] is good. Their data for $Ra^{0.333}/(Re^{0.8} \cdot Pr^{0.4}) < 0.25$ were eliminated since they do not show the deterioration in Nusselt number similar to the data of all other authors with the exception of Metais [26]. Only the data of Saylor and Joye [16] do not match the correlation for the natural convection limiting case. In the region of mixed convection, $0.05 < Ra^{0.333}/(Re^{0.8} \cdot Pr^{0.4}) < 0.25$, see the inset in Fig. 8, it can be stated that the increasing L/d-ratio, the minimum of the related Nusselt number moves down and to smaller $Ra^{0.333}/(Re^{0.8} \cdot Pr^{0.4})$, which means that the influence of natural convection is stronger.

For opposing mixed convection, there is no significant impact of L/d on the related Nusselt number except for the natural convection limiting case, which behaves the same way as aiding flow. For unknown reasons the data of Saylor and Joye do not fit to the new correlation. The solid line shows the new correlation which will be presented in the following part.

4.3. *New correlations for mixed convection heat transfer*

Two new correlations are proposed to predict Nusselt numbers for opposing and aiding mixed natural and forced convection heat transfer. They are superpositions of pure natural and pure forced convection heat transfer correlations taking into account the laminarization of turbulent flows in vertical tubes under aiding influence of natural convection.

For opposing mixed convection, a simple superposition of the pure natural and the pure forced convection limiting case yields useful results. The

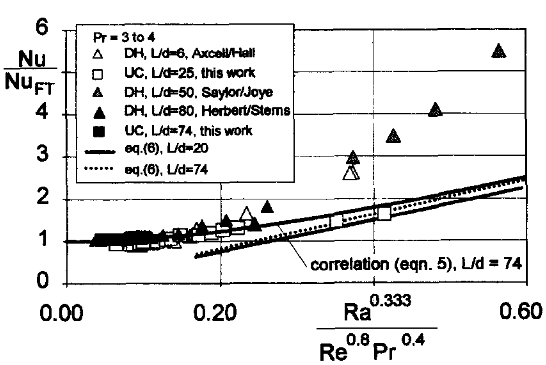

Fig. 9. Nusselt numbers related to Nusselt number for pure forced convection plotted over the parameter $Ra^{0.333}/(Re^{0.8} \cdot Pr^{0.4})$ for opposing mixed convection flow in vertical tubes with respect to L/d ratio. Data were taken from this work and from literature [14–16]. $Pr \cong 3, 5$.

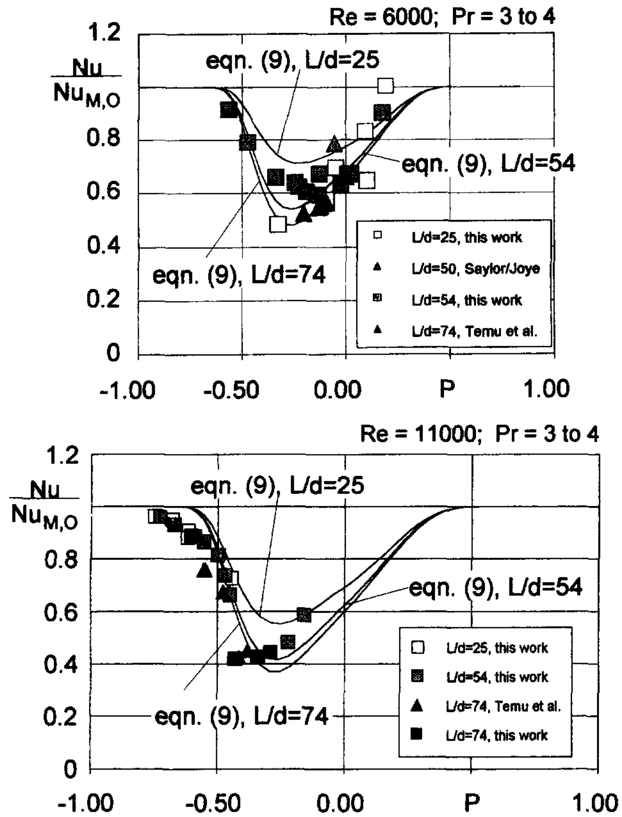

Fig. 10. New correlation to describe aiding mixed convection heat transfer.

exponent has been obtained by a least square fit to the experimental results presented in Figs. 7 and 9 of this work.

$$Nu_{\mathrm{M,O}} = \sqrt[2]{Nu_{\mathrm{FT}}^2 + Nu_{\mathrm{NT}}^2} \tag{5}$$

The Nusselt number for pure forced turbulent convection heat transfer can be obtained by the equations (2) or (3) as cited above. For pure natural convection the constant in the equation of Churchill and Chu [28] has been fitted by a least square fit to the data for dominant natural convection, i.e. $Ra^{0.333}/(Re^{0.8} \cdot Pr^{0.4}) > 0.2$, of [11, 12, 14–17] and of this work

$$Nu_{\mathrm{NT}} = 0.122 \cdot Ra^{0.333} \cdot \left(1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right)^{-16/27}. \tag{6}$$

The fluid properties were calculated at the tube-side bulk temperature, which is the arithmetic mean of inlet and outlet fluid temperature, except for the Rayleigh number which uses film temperature. As characteristic length, the inner diameter of the tube was used. To prove the good agreement with experimental results, the correlation has been added to Fig. 9.

For aiding mixed convection, a correlation has been arrived at by a new method to display the experimental data, see Fig. 10(a) and (b). On the ordinate the Nusselt number is plotted related to the Nusselt number calculated for the same Reynolds, Prandtl and Rayleigh numbers under opposing mixed convection regime following the aforementioned correlation, see equation (5):

$$\frac{Nu_{\mathrm{M,A}}}{Nu_{\mathrm{M,O}}}. \tag{7}$$

Theoretically, the related Nusselt number ranges from 0 to 1, never dropping below 0.3. On the abscissa, the difference between Nusselt numbers calculated for pure natural and pure forced convection limiting cases is plotted. In order to do so, the above mentioned correlations were used with the Reynolds, Prandtl and Rayleigh numbers of the case under consideration. This Nusselt number difference is related to the Nusselt number calculated for opposing mixed convection, just the same way as in the procedure used for the ordinate.

$$P = \frac{Nu_{\mathrm{NT}} - Nu_{\mathrm{FT}}}{Nu_{\mathrm{M,O}}}. \tag{8}$$

This ratio ranges from -1 for pure forced convection

Table 3. Parameters in the new correlation, equation (9)

a	b	c
2.0	1.3	-0.5

Table 4. Valid ranges of the new correlation, equation (9)

Combination	Reynolds number	Rayleigh number	Prandtl number
Aiding flow	3000–60 000	$6 \cdot 10^6$–$4 \cdot 10^8$	0.7–5.1
Opposing flow	3000–120 000	$3 \cdot 10^7$–$1 \cdot 10^9$	0.7–5.0

to $+1$ for pure natural convection regime. In Fig. 10(a) and (b), the related Nusselt numbers for aiding mixed convection heat transfer are shown schematically according to this new method for tube-side Reynolds numbers of 6000 and 11 000. The curve shows the characteristic decline in related Nusselt numbers to about 0.4 for the mixed convection regime, i.e. $-0.5 < P < 0$. The behavior of the related Nusselt numbers in Fig. 10(a) and (b) can be described by the Gauss equation. This leads to the following correlation

$$Nu_{\mathrm{M,A}} = Nu_{\mathrm{M,O}} \cdot \left\{1 - \left(1 - a \cdot \frac{Nu_{\mathrm{FL}}}{Nu_{\mathrm{M,O}}}\right) \cdot f(P)\right\} \quad (9)$$

with

$$f(P) = \exp\left[-b \cdot \left(\frac{P}{1-|P|} - c\right)^2\right].$$

The Nusselt number for pure forced laminar convection is calculated according to equation (4). Equation (9) contains three parameters, a, b and c which were adapted to the experimental data shown in Figs. 6 and 8 by means of a least square fit. Their values are listed in Table 3. The Reynolds numbers, the Rayleigh numbers and the Prandtl numbers for which both correlations have been fitted to experimental results are given in Table 4.

5. CONCLUSIONS

(1) Our experiments confirm that heat and mass flux directions have no significant impact on mixed convection heat transfer in vertical turbulent tube flows.

(2) In aiding mixed convection, increase of L/d leads to a decrease in heat transfer occurring at smaller $Ra^{0.333}/(Re^{0.8} \cdot Pr^{0.4})$. In opposing mixed convection, there seems to be no influence of the L/d ratio on the heat transfer for $6 < L/d < 80$.

(3) For Reynolds numbers from 3000 to 120 000, Rayleigh numbers from $3 \cdot 10^7$ to $1 \cdot 10^9$, and Prandtl numbers from 0.7 to 5.0 opposing mixed convection heat transfer can be predicted by equation (5).

(4) For Reynolds numbers from 3000 to 60 000, Rayleigh numbers from $6 \cdot 10^6$ to $4 \cdot 10^8$, and Prandtl numbers from 0.7 to 5.1 aiding mixed convection heat transfer can be predicted by equation (9).

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